Name Date Time

PROJECT 2

Deficient, Abundant, and Perfect Numbers

A **factor** of a whole number *N* is any whole number that can be multiplied by a whole number to give *N* as the product. For example, 5 is a factor of 30 because 6 ∗ 5 = 30. Also, 6 is a factor of 30. Every whole number has itself and 1 as factors.

A **proper factor** of a whole number is any factor of that number except the number itself. For example, the *factors* of 10 are 1, 2, 5, and 10. The *proper factors* of 10 are 1, 2, and 5.

A whole number is a **deficient number** if the sum of its proper factors is less than the number. For example, 10 is a deficient number because the sum of its proper factors is 1 + 2 + 5 = 8, and 8 is less than 10.

A whole number is an **abundant number** if the sum of its proper factors is greater than the number. For example, 12 is an abundant number because the sum of its proper factors is 1 + 2 + 3 + 4 + 6 = 16, and 16 is greater than 12.

A whole number is a **perfect number** if the sum of its proper factors is equal to the number. For example, 6 is a perfect number because the sum of its proper factors is 1 + 2 + 3 = 6.

Exploration

List the proper factors of each number from 1 to 50 in the table on *Math Masters,* pages 380 and 381. Then find the sum of the proper factors of each number, and record it in the third column of the table. Finally, make a check mark in the appropriate column to show whether the number is deficient, abundant, or perfect.

Divide the work with the other members of your group. Have partners use factor rainbows to check each other's work. When you are satisfied that all the results are correct, answer the questions on page 381.

Name Date Time

PROJECT 2

Deficient, Abundant, and Perfect Numbers *cont.*

Number	Proper Factors	Sum of Proper Factors	Deficient	Abundant	Perfect
1		0	✓		
2					
3					
4					
5					
6	1, 2, 3	6			✓
7					
8					
9					
10	1, 2, 5	8	✓		
11					
12	1, 2, 3, 4, 6	16		✓	
13					
14					
15					
16					
17					
18					
19					
20					
21					
22					
23					
24					
25					
26					
27					
28					
29					
30					
31					
32					
33					
34					

Name Date Time

PROJECT 2

Deficient, Abundant, and Perfect Numbers *cont.*

Number	Proper Factors	Sum of Proper Factors	Deficient	Abundant	Perfect
35					
36					
37					
38					
39					
40					
41					
42					
43					
44					
45					
46					
47					
48					
49					
50					

Source: *The Math Teacher's Book of Lists.* San Francisco: Jossey-Bass, 2005.

Refer to the results in your table.

1. What are the perfect numbers up to 50? ______

2. Is there an abundant number that is not an even number? ______

3. Are all deficient numbers odd numbers? ______

4. What is the next number greater than 50 for which the sum of its proper factors is 1? ______

5. The sum of the proper factors of 4 is 1 less than 4. List all the numbers through 50 for which the sum of the proper factors is 1 less than the number itself. ______

6. What do you think is the next number greater than 50 for which the sum of its proper factors is 1 less than the number itself? ______

Name Date Time

PROJECT 2

A Perfect-Number Challenge

Perfect numbers become big very quickly. The third perfect number has 3 digits, the fourth has 4 digits, the fifth has 8 digits, the sixth has 10 digits, and the thirty-second has 455,663 digits! In other words, perfect numbers are hard to find.

You can find perfect numbers without having to find the sum of the proper factors of every number. Here is what you do:

1. Complete the pattern of starting numbers in the first column of the table.
2. List the factors of each starting number in the second column.
3. Write the sum of the factors of each starting number in the third column.
4. If the sum of the factors of the starting number is prime, multiply this sum by the starting number itself. The product is a perfect number. Record it in the last column.

The first perfect number is 6. Try to find the next three perfect numbers.

Starting Number	Factors	Sum of Factors	Perfect Number
2	1, 2	3	6
4			
8			

People have been fascinated by perfect numbers for centuries. The ancient Greeks knew the first four. No one found the fifth perfect number until the year 1456. Computers now carry on the search for perfect numbers. When this book went to press, 42 perfect numbers had been identified. All the perfect numbers found so far are even numbers.