

The Search for Prime Numbers



You probably know the following definitions of prime and composite numbers:

A **prime number** is a whole number that has exactly two **factors**. The factors are 1 and the number itself. For example, 7 is a prime number because its only factors are 1 and 7. A prime number is divisible by only 1 and itself.

A **composite number** is a whole number that has more than two factors. For example, 10 is a composite number because it has four factors: 1, 2, 5, and 10. A composite number is divisible by at least three whole numbers.

The number 1 is neither prime nor composite.

For centuries, mathematicians have been interested in prime and composite numbers because they are the building blocks of whole numbers. They have found that every composite number can be written as the product of prime numbers. For example, 18 can be written as 2 * 3 * 3.

Around 300 B.C., the Greek mathematician Euclid (yOO'klid) proved that there is no largest prime number. No matter how large a prime number you find, there will always be larger prime numbers. Since then, people have been searching for more prime numbers. In 1893, a mathematician was able to show that there are more than 50 million prime numbers between the numbers 1 and 1 billion.

The Greek mathematician Eratosthenes (ĕr´ə-tŏs´ thə-nēz´), who lived around 200 в.с., devised a simple method for finding prime numbers. His strategy was based on the fact that every **multiple of a number** is divisible by that number. For example, the numbers 2, 4, 6, 8, and 10 are multiples of 2, and each of these numbers is divisible by 2. Here is another way to say it: A whole number is a factor of every one of its multiples. For example, 2 is a factor of 2, 4, 6, 8, and 10. The number 2 has only one other factor, the number 1, so 2 is a prime number. All other multiples of 2 are composite numbers.

Eratosthenes' method is called the **Sieve of Eratosthenes**. The directions for using the sieve to find prime numbers are given on *Math Masters*, page 375.

Since the time of Eratosthenes, mathematicians have invented more powerful methods for finding prime numbers. Some methods use formulas. Today, people use computers. The largest prime number known when this book went to press had 9,152,052 digits. If that number were printed in a book with pages the same size as this page, in the same size type, the book would be about 1,400 pages long.



The Sieve of Eratosthenes



Follow the directions below for *Math Masters*, page 376. When you have finished, you will have crossed out every number from 1 to 100 that is not a prime number.

- 1. Because 1 is not a prime number, cross it out.
- 2. Circle 2 with a colored marker or crayon. Then count by 2, crossing out all multiples of 2; that is, 4, 6, 8, 10, and so on.
- 3. Circle 3 with a different colored marker or crayon. Cross out every third number after 3—6, 9, 12, and so on. If a number is already crossed out, make a mark in a corner of the box. The numbers you have crossed out or marked are multiples of 3.
- **4.** Skip 4, because it is already crossed out, and go on to 5. Use a new color to circle 5, and cross out multiples of 5.
- 5. Continue in the same pattern. Start each time by circling the next number that is not crossed out. Cross out all multiples of that number. If a number is already crossed out, make a mark in a corner of the box. Use a different color for each new set of multiples.
- **6.** Stop when there are no more numbers to be circled or crossed out. The circled numbers are the prime numbers from 1 to 100.

7.	List all the prime numbers from 1 to 100.

PROJECT 1

The Sieve of Eratosthenes continued



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	.74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



The Sieve of Eratosthenes continued



1.	What are the crossed-out numbers greater than 1 called?						
2.	Notice that 6 is a multiple of both 2 and 3. Find two other numbers that are multiples of both 2 and 3.						
3.	Find a number that is a multiple of 2, 3, and 5. (Hint: Look at the colors.)						
4.	Find a number that is a multiple of 2, 3, 4, and 5.						
5.	Choose any crossed-out number between 50 and 60. List its factors.						
6.	List the crossed-out numbers that have no marks in the corners of their boxes.						
7.	Find a pair of consecutive prime numbers.						
	Are there any others? If yes, list them.						
8.	The numbers 3 and 5 are called twin primes because they are separated by just one composite number. List all the other twin primes from 1 to 100.						
9.	Why do you think this grid is called a sieve?						

PROJECT

The Sieve of Eratosthenes continued



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101	102	103	104	105	106	107	108	109	100
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200